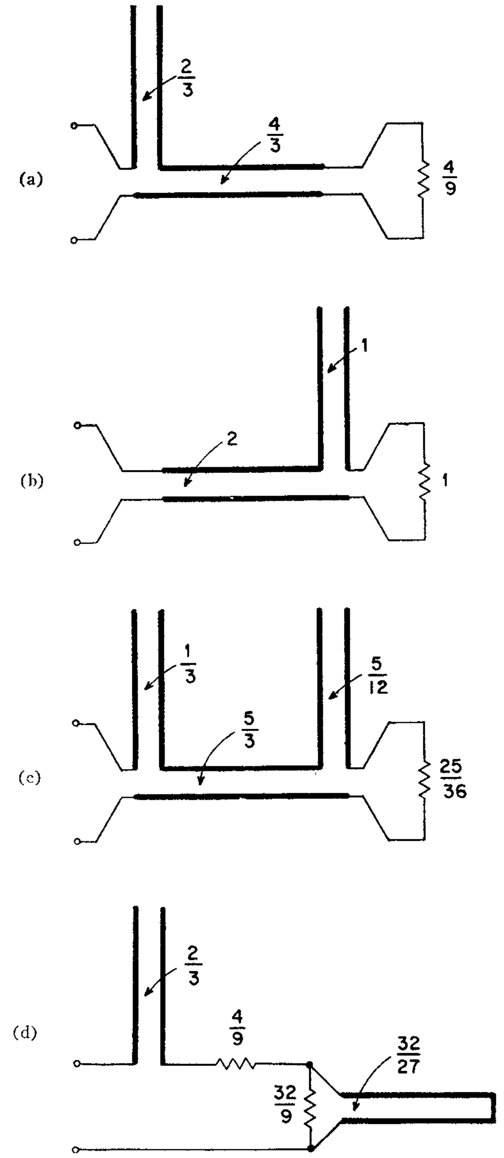


Fig. 1—Four circuits with the same input impedance (1). All lines are of length $\theta$; all numbers are either resistances or characteristic impedances of lines.

The theorem thus stated specifically excludes common factors of $(m_1+n_1)$ and $(m_2+n_2)$. As Riblet has pointed out,[2] this results in a mathematical loss of generality, which however is physically trivial, since no measurement can distinguish between two one-ports consisting, respectively, of a resistance $R$, and a resistance $R$ preceded by a length of transmission line of characteristic impedance $R$.

A final comment on terminology. In my paper[5] on the same subject, I specified "homogeneous" transformers without clearly defining this term, and would like to make up this omission now.

Definition: A homogeneous waveguide is one in which the guide wavelength is independent of position.

As a corollary, an inhomogeneous waveguide is one in which the guide wavelength varies with position; *i.e.*, it is not uniformly dispersive. The quarter-wave transformers which are the subject of this discussion and of the references mentioned so far are all homogeneous transformers. There has been little need in the past to distinguish between homogeneous and inhomogeneous quarter-wave transformers (as defined above), as no theory existed for the latter. However, inhomogeneous quarter-wave transformers have recently been analyzed,[6] and the above terminology was introduced to distinguish between these two situations.

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[5] L. Young, "Tables for cascaded homogeneous quarter-wave transformers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 233–237; April, 1959.

[6] L. Young, "Design of microwave stepped transformers with applications to filters," Dr. Eng. dissertation, The Johns Hopkins University, Baltimore, Md.; 1959.

## Broad-Band Stub Design*

Mr. Muehe's results for broad-band stubs,[1] while going beyond that of previous investigators, are similar to my design curves published earlier without detailed derivation.[2] The main difference is that my curves were based on a formula involving the bandwidth as defined by the lowest and highest frequencies of *zero* reflection. Mr. Muehe's analysis is therefore superior to my simpler approach; my design curves erred on the safe side in predicting a slightly smaller bandwidth.

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* Received by the PGMTT, May 15, 1959.

[1] C. E. Muehe, "Quarter-wave compensation of resonant discontinuities," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 296–297; April, 1959.

[2] L. Young, "Coaxial stub design," *Electronics*, vol. 30, p. 188; July 1, 1957.

## Attenuation of the $HE_{11}$ Mode in the *H*-Guide*

The properties of the lowest order hybrid mode of the *H*-guide line have been analyzed by Tischer in a number of papers.[1–3] This type of transmission line has also been under investigation at this laboratory for some time and a number of discrepancies exist between our analysis and those of Tischer.

The geometry of this line is shown in Fig. 1. For comparison purposes, the coordinates and notation introduced by Tischer will be used throughout this letter.

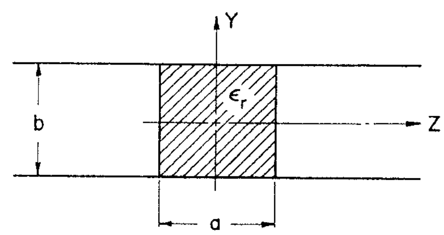


Fig. 1—Cross section of the *H*-guide line. The positive $x$-axis is into the paper.

In Tischer[1,2] the dielectric slab is considered thin and as a result, 1) dielectric losses are neglected, and 2) in the derivation of the expression for the wall losses the power flow in the dielectric slab and the losses in the portion of the walls contacting

* Received by the PGMTT, May 15, 1959.

[1] F. J. Tischer, "Microwellenleitung mit geringen verlusten" (Waveguides with small losses), *Arch. elekt. Uebertragung*, vol. 7, pp. 592–596; December, 1953.

[2] F. J. Tischer, "The *H*-guide, a waveguide for microwaves," 1956 IRE CONVENTION RECORD, pt. 5, pp. 44–47.

[3] F. J. Tischer, "Properties of the *H*-guide at microwaves and millimeter waves," 1958 WESCON CONVENTION RECORD, pt. 1, pp. 4–12.

the dielectric slab are neglected. The resulting equation for the attenuation due to the loss in the walls is a good approximation for a thin slab but it is not generally correct.

The analysis conducted at this laboratory yields the following equations for the attenuation due to the wall and dielectric loss respectively:

$$\alpha_w = \frac{\dfrac{2R_s}{bZ_0}\pi^3\left(\dfrac{a}{\lambda_0}\right)^3\left(\dfrac{\lambda_0}{2b}\right)^2\epsilon_r}{\sqrt{\pi^2\left(\dfrac{a}{\lambda_0}\right)^2\left[\epsilon_r-\left(\dfrac{\lambda_0}{2b}\right)^2\right]-\left(k_d\dfrac{a}{2}\right)^2}} \cdot \left\{\frac{k_d\dfrac{a}{2}\tan k_d\dfrac{a}{2}+\sin^2 k_d\dfrac{a}{2}+\epsilon_r\cos^2 k_d\dfrac{a}{2}}{\left[\pi^2\epsilon_r\left(\dfrac{a}{\lambda_0}\right)^2-\left(k_d\dfrac{a}{2}\right)^2\right]k_d\dfrac{a}{2}\tan k_d\dfrac{a}{2}+\pi^2\epsilon_r\left(\dfrac{a}{\lambda_0}\right)^2\left(\sin^2 k_d\dfrac{a}{2}+\epsilon_r\cos^2 k_d\dfrac{a}{2}\right)}\right\} \quad (1)$$

$$\alpha_d = \frac{\left[\pi^2\epsilon_r\left(\dfrac{a}{\lambda_0}\right)^2-\left(k_d\dfrac{a}{2}\right)^2\right]\tan\delta}{a\sqrt{\pi^2\left(\dfrac{a}{\lambda_0}\right)^2\left[\epsilon_r-\left(\dfrac{\lambda_0}{2b}\right)^2\right]-\left(k_d\dfrac{a}{2}\right)^2}} \cdot \left\{\frac{\pi^2\epsilon_r\left(\dfrac{a}{\lambda_0}\right)^2k_d\dfrac{a}{2}\tan k_d\dfrac{a}{2}+\left[\pi^2\epsilon_r\left(\dfrac{a}{\lambda_0}\right)^2-2\left(k_d\dfrac{a}{2}\right)^2\right]\sin^2 k_d\dfrac{a}{2}}{\left[\pi^2\epsilon_r\left(\dfrac{a}{\lambda_0}\right)^2-\left(k_d\dfrac{a}{2}\right)^2\right]k_d\dfrac{a}{2}\tan k_d\dfrac{a}{2}+\pi^2\epsilon_r\left(\dfrac{a}{\lambda_0}\right)^2\left(\sin^2 k_d\dfrac{a}{2}+\epsilon_r\cos^2 k_d\dfrac{a}{2}\right)}\right\}, \quad (2)$$

where $\alpha_w$ and $\alpha_d$ are the wall and dielectric attenuation foactors in nepers/meter if $a$, $b$, and $\lambda_0$ are in meters, $R_s$ is the surface resistance of the conducting walls, $\tan\delta$ is the loss tangent of the dielectric slab, $Z_0$ is the impedance of free space, and $k_d$ is the transverse wave number for the $z$-direction in the dielectric filled region. $k_d$ is related to the dielectric constant ($\epsilon_r$), the dielectric slab width ($a$), and the free space wavelength ($\lambda_0$) by the following transcendental equation:

$$\left(k_d\frac{a}{2}\right)^2\left[\frac{\tan^2 k_d\dfrac{a}{2}}{\epsilon_r^2}+1\right] = (\epsilon_r-1)\pi^2\left(\frac{a}{\lambda_0}\right)^2. \quad (3)$$

Eq. (3) is obtained by satisfying the boundary conditions at the dielectric-air interfaces and requiring that the axial propagation constant be the same in the air and dielectric regions. Graphical techniques have been used to determine $[k_d(a/2)]$ as a function of $(a/\lambda_0)$ and $\epsilon_r$. For the lowest hybrid mode ($HE_{11}$), $[k_d(a/2)]$ is constrained to be within the following interval.

$$0 \le \left(k_d\frac{a}{2}\right) \le \frac{\pi}{2}.$$

If $(\epsilon_r-1)\pi^2(a/\lambda_0)^2$ is very small, then $[k_d(a/2)]$ becomes a very small angle and the following approximation may be used for (3).

$$\left(k_d\frac{a}{2}\right)^2 \approx (\epsilon_r-1)\pi^2(a/\lambda_0)^2. \quad (4)$$

Using (4), the following thin slab limiting value for $\alpha_w$ and thin slab approximation for $\alpha_d$ are obtained.

$$\lim_{a\to 0}\alpha_w = \frac{2R_s}{bZ_0}\cdot\frac{(\lambda_0/2b)^2}{\sqrt{1-(\lambda_0/2b)^2}} \quad (5)$$

$$\alpha_d = \frac{2\pi^3(\epsilon_r-1)(a/\lambda_0)^2}{\epsilon_r^2\lambda_0\sqrt{1-(\lambda_0/2b)^2}}\tan\delta,$$

$$\text{for } (\epsilon_r-1)\pi^2\left(\frac{a}{\lambda_0}\right)^2 \ll 1. \quad (6)$$

It is of interest to comment on the other extreme condition in which $(\epsilon_r-1)\pi^2(a/\lambda_0)^2$ is very large and $[k_d(a/2)]$ approaches $\pi/2$. In that case (3), (1) and (2) become, respectively,

$$\left(k_d\frac{a}{2}\right)^2\tan^2 k_d\frac{a}{2} \approx \epsilon_r^2(\epsilon_r-1)\pi^2(a/\lambda_0)^2 \quad (7)$$

$$\lim_{a\to\infty}\alpha_w = \frac{2R_s}{bZ_0}\cdot\frac{(\lambda_0/2b)^2}{\sqrt{\epsilon_r-(\lambda_0/2b)^2}} \quad (8)$$

$$\lim_{a\to\infty}\alpha_d = \frac{\pi\epsilon_r}{\lambda_0\sqrt{\epsilon_r-(\lambda_0/2b)^2}}\tan\delta. \quad (9)$$

It should be noted that (5) and (8) are, respectively, the expressions for the wall loss attenuation of a completely air-filled and a completely dielectric-filled parallel plane transmission line in which the lowest order TE mode is propagating. This serves as a good check on the validity of (1), since the $HE_{11}$ mode approaches the field configuration of this TE mode when the space between the conducting walls is uniformly loaded.

If the same thin slab and thick slab approximations are applied to the equations for $\alpha_w$ in Tischer,[1,2] it is found that the thin slab approximations agree but the thick slab approximations do not.

In a more recent publication,[3] Tischer outlines a method of deriving equations for $\alpha_w$ and $\alpha_d$, in which the dielectric slab need not be thin. The final expression for $\alpha_w$, which he presents, does not agree with (1). Furthermore if the thin and thick slab approximations are applied to his equation, it is found that they do not agree with either the limiting expressions presented here [(5) and (8)] or with those determined from his earlier publications.[1,2] In addition, the various curves of $\alpha_w$ as a function of slab width in Tischer's most recent publication[3] do not agree with each other. In this most recent work of Tischer's, no equation is presented for the attenuation due to the loss in the dielectric; however, a curve of $\alpha_d$ vs slab width is presented for a particular line. This curve does not agree with the corresponding curve computed from (2). The two curves of $\alpha_d$ vs slab width are shown here in Fig. 2. Also shown here is a curve of $\alpha_w$ as computed from (1).

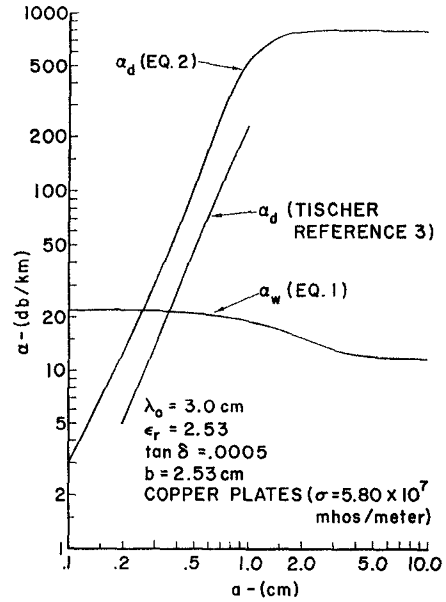


Fig. 2—Attenuation due to wall loss and dielectric loss as a function of dielectric slab width.

These curves show that except for very thin dielectric slabs (assuming typical values for $\tan\delta$) the dielectric loss is the principal contributor to the total attenuation. Eq.s (2) and (6) show that, if either very thin slabs or very low dielectric constant materials are used, the dielectric loss can be comparable to or less than the very low wall loss. In that case there will be a range of frequencies in which the highly desirable property of decreasing attenuation with increasing frequency will exist. The price for this advantage is that the fields in the air space will decay very slowly with increasing distance from the dielectric slab. Therefore, the conducting planes will have to be extended in the $z$-direction.

The $HE_{11}$ mode on this line has also been analyzed by Moore and Beam.[4] Their equations for the field components of this mode are erroneous, however, in that they do not show that the transverse magnetic field which is parallel to the conducting planes is everywhere zero. This fact had been reported by Tischer and was confirmed by this writer.

It should be noted that although the $HE_{11}$ mode is the lowest order hybrid mode of this line, it is not the dominant mode.

[4] R. A. Moore and R. E. Beam, "A duo-dielectric parallel plane waveguide," *Proc. NEC*, vol. 12, pp. 689–705; April, 1957.

There is a class of TE modes which can propagate on this line.[5,6] The $TE_{10}$ mode is the dominant mode of this structure. These TE modes should not be confused with the prior-mentioned TE modes which can only propagate in a uniformly-loaded parallel-plane line.

Tischer has shown that a conducting plane can be placed at the plane $z=0$ and not affect the field configuration of the $HE_{11}$ mode. This conducting plane completely suppresses all of the TE modes which are symmetric about the $z=0$ plane ($TE_{10}$, $TE_{30}$, etc.). If in addition the slab width is such that

$$\frac{a}{\lambda_0} < \frac{1}{2\sqrt{\epsilon_r - 1}},$$

the $TE_{20}$ and all higher order antisymmetric TE modes cannot propagate. The $HE_{11}$ mode will then be the dominant mode of the resulting trough line. The attenuation due to the loss in this added conducting plane will not, however, decrease with increasing frequency.

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[5] M. Cohn, "Parallel plane waveguide partially filled with a dielectric," PROC. IRE, vol. 46, pp. 1952–1953; December, 1958.

[6] M. Cohn, "Propagation in a dielectric-loaded parallel plane waveguide," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 202–208; April, 1959.

## *Author's Comment*[7]

It was interesting to learn that my proposal of 1952 for a new waveguide, the *H*-guide, found such attention, and that it became the subject of investigation at several places.

Concerning discrepancies mentioned in the above letter of Cohn, it is suggested that one should consider that approximate solutions, in general, depend on the neglections introduced and may not be called either correct or incorrect.

The equations for the attenuation of the *H*-guide dealt with in the above letter are quite complicated. One part of the attenuation constant, contributed by the wall losses, follows for comparison. The neglections, on which the approximations are based, are indicated.

The equations are valid for the waveguide of infinite height which is excited in the fundamental hybrid mode. They are approximations based on the field distribution in a lossless guide:

$$\alpha_w = \frac{R_s\left(\frac{\omega\epsilon_0\pi}{b\Gamma k_a}\right)^2}{\frac{b}{2}\left(\frac{\omega\epsilon_0}{\Gamma}\right)\left(1+\frac{\omega^2\epsilon_0\mu_0}{k_a^2}\right)} m; \qquad (1)$$

$$m = \frac{\cos^2\psi + \frac{k_a}{k_d}[\psi + \sin\psi\cos\psi]}{\cos^2\psi + \frac{k_a}{\epsilon_r k_d}[\psi + \sin\psi\cos\psi]}, \qquad (2)$$

where

$$\psi = k_d \frac{a}{2}.$$

For small values of $a$, where the wall currents and the Poynting vector become approximately independent of $z$ inside the dielectric (varying with cos $k_d z$, where $k_d z \ll 1$), $m \to 1$, and it is the same as if the losses in the walls inside the dielectric and the power transmitted in this region were neglected.

Eq. (1) becomes, after transformation,

$$\alpha_w = \frac{2R_s}{bZ_0}\left(\frac{\lambda_0}{2b}\right)^2 \frac{1}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_{c0}}\right)^2}} \cdot \left[1+\left(\frac{k_a\lambda_0}{2\pi}\right)^2\right]^{-1} \cdot \left[1+\left(\frac{k_a\lambda_0}{2\pi}\right)^2\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2\right]^{-1/2}, \qquad (3)$$

which is (16) of the publication cited by Cohn.[3]

A limiting value for $\alpha_w = \alpha_0$ is obtained if $a \to 0$.

$$\alpha_0 = \frac{2R_s}{bZ_0}\left(\frac{\lambda_0}{2b}\right)^2 \frac{1}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_{c0}}\right)^2}}, \qquad (4)$$

which is the attenuation of the rectangular waveguide of infinite height excited in the $TE_{10}$ mode. Typographical errors occurred in the above cited publication in the exponents of the equations for $\alpha_0$ and (16).

Discrepancies seem still to exist between Cohn's (1) and the above (1) and (2), but no errors could be detected in our equation for $\alpha_w$.

Due to lack of time and manpower in our case, I have to leave the honor of investigating and detecting this discrepancy to somebody else.

Mr. Chen Pang Wu's assistance in the calculations and checks is appreciated.

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[7] Received by the PGMTT, June 22, 1959.

## *Mr. Cohn's Reply*[8]

In Dr. Tischer's above letter, I am in complete agreement with his corrected (3), since it is clearly labeled as an approximation for a thin dielectric slab. With regard to his more general equations, (1) and (2), discrepancies between our results certainly do exist. I submit that it is a necessary (though not sufficient) condition that the limiting values of $\alpha_w$ for $a \to 0$ and $a \to \infty$ should agree respectively with the well-known solutions for the attenuation of an infinitely-high rectangular waveguide which is air filled and dielectric filled. Eqs. (1) and (2) in Dr. Tischer's letter do not satisfy the thick slab limiting condition.

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[8] Received by the PGMTT, June 25, 1959.

# Experimental Determination of Wavelength in Dielectric-Filled Periodic Structures*

Let it be required to determine the guided wavelength in a dielectric-filled periodic structure, such as a corrugated wall or serrated waveguide. The accepted travelling probe technique requires a slot in the broad wall of the guide and a groove in the dielectric material. Even if the errors introduced by these modifications could be tolerated, other effects render this technique unsuitable. One of these is a surface-wave effect which results in a measured wavelength higher than the one in the guide and lower than the free-space value. If the structure is dissipative, such as a serrated guide, more difficulties arise.

A suggested solution to this experimental problem is based on Deschamps' method of determining the elements of the scattering matrix.[1] All that is needed is the value of the argument of the transfer coefficient $S_{12}e^{i\theta}$ where $\theta+2\pi n$ is the electrical length of the sample and $n$ is an integer.

Suppose now that $\theta$ has been measured at two frequencies, $f$ and $f+\delta f$, where $\delta f/f$ is of the order of $10^{-3}$. Let the guided wavelengths in the two measurements be $\lambda_g$ and $\lambda_g+\delta\lambda_g$, respectively, and let $L$ be the length of the sample. Then,

$$\lambda_g(\theta + 2\pi n) = 2\pi L \qquad (1a)$$

$$(\lambda_g + \delta\lambda_g)(\theta + \delta\theta + 2\pi n) = 2\pi L \qquad (1b)$$

Implied in (1b) is the fact that $\theta$ is a continuous function of $f$ which is true for periodic structures whose period is small compared with the guided wavelength. It appears that we have on hand two second-order equations in three unknowns: $n$, $\lambda_g$ and $\delta\lambda_g$. But by expressing $\delta\lambda_g$ in terms of $\lambda_g$, $f$, and $\delta f$ we can obtain one third order equation in $n$. It is expedient to use a method of numerical solution, because once an approximate solution has been arrived at, the exact value of $n$ becomes the nearest integer.

We represent the structure by an equivalent dielectric constant $\epsilon$ and write

$$\lambda_g = \frac{\lambda}{\sqrt{\epsilon - \frac{\lambda^2}{\lambda_c^2}}} \qquad (2)$$

$$\lambda = \frac{c}{f} \qquad (3)$$

where $\lambda$ and $\lambda_c$ are the free-space and cut-off wavelength in the unloaded guide respectively.

To a first approximation we have

$$\delta\lambda_g = \frac{d\lambda_g}{d\lambda}\cdot\frac{d\lambda}{df}\cdot\delta f. \qquad (4)$$

From (2) and (3) we have

$$\frac{d\lambda_g}{d\lambda} = \left(\epsilon - \frac{\lambda^2}{\lambda_c^2}\right)^{-1/2} + \frac{\lambda^2}{\lambda_c^2}\left(\epsilon - \frac{\lambda^2}{\lambda_c^2}\right)^{-3/2}$$

and

* Manuscript received by the PGMTT, April 24, 1959; revised manuscript received, May 21, 1959.

[1] G. A. Deschamps, "Determination of reflection coefficients and insertion loss of a waveguide junction," *J. Appl. Phys.*, vol. 24, pp. 1046–1050; August, 1953.